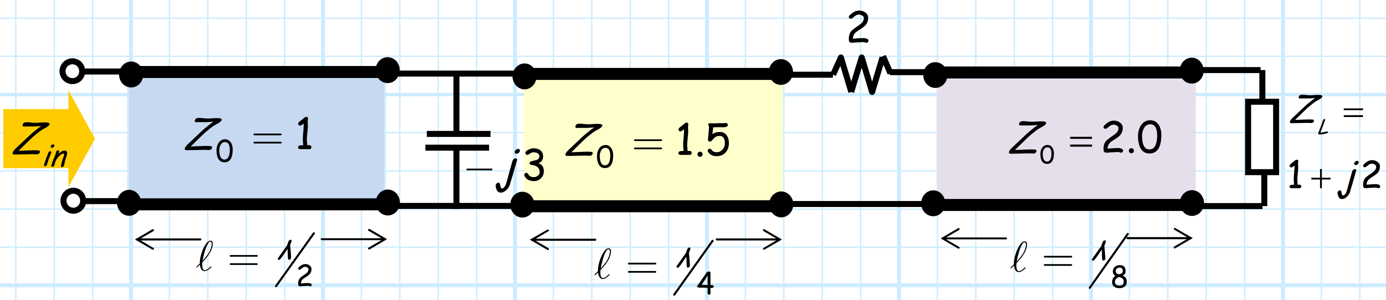
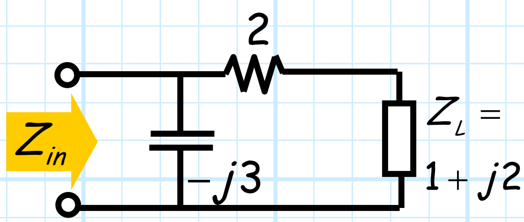
Example: Input Impedance

Consider the following circuit:



If we **ignored** our new μ-wave knowledge, we might **erroneously** conclude that the input impedance of this circuit is:



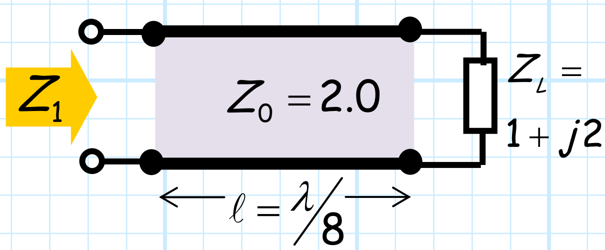
Therefore:

$$Z_{in} = \frac{-j3(2+1+j2)}{-j3+2+1+j2} = \frac{6-j9}{3-j} = 2.7 - j2.1$$

Of course, this is **not** the correct answer!

We must use our **transmission line theory** to determine an accurate value.

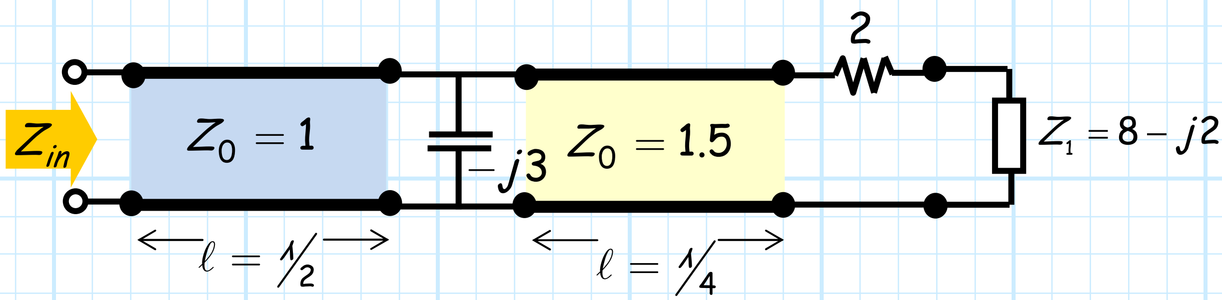
Define Z_1 as the input impedance of the last section:



we find that Z_1 is :

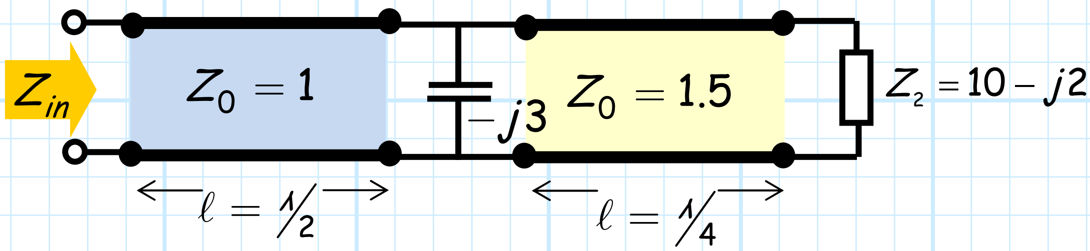
$$\begin{aligned} Z_1 &= Z_0\left(\frac{Z_L\cos\beta\ell + j\,Z_0\sin\beta\ell}{Z_0\cos\beta\ell + j\,Z_L\sin\beta\ell}\right) \\ &= 2\left(\frac{(1+j2)\cos\left(\pi/4\right) + j\,2\sin\left(\pi/4\right)}{2\cos\left(\pi/4\right) + j(1+j2)\ \sin\left(\pi/4\right)}\right) \\ &= 2\left(\frac{1+j4}{j}\right) \\ &= 8 - j2 \end{aligned}$$

Therefore, our circuit now becomes:

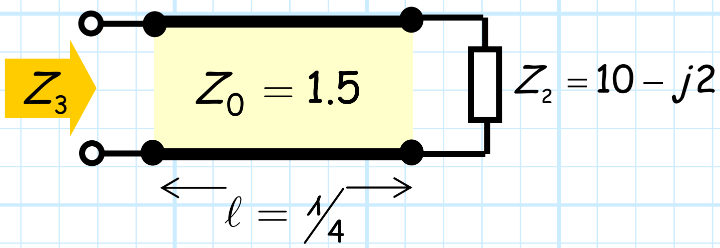


Note the resistor is in **series** with impedance Z_1. We can **combine** these two into one impedance defined as Z_2:

$$Z_2 = 2 + Z_1 = 2 + (8 - j2) = 10 - j2$$



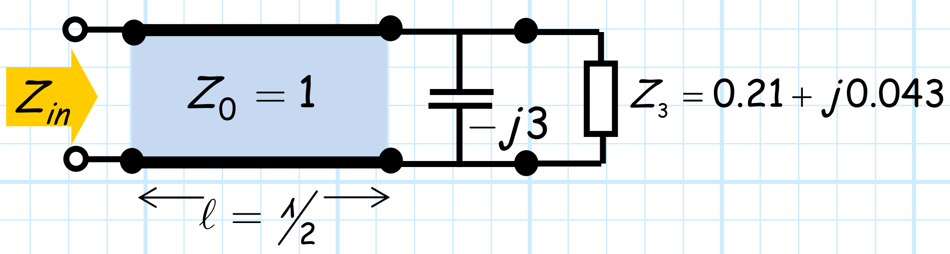
Now let's define the input impedance of the **middle** transmission line section as Z_3:



Note that this transmission line is a **quarter wavelength** ($\ell = \lambda/4$). This is one of the **special** cases we considered earlier! The input impedance Z_3 is:

$$
\begin{aligned}
Z_3 &= \frac{Z_0^{\,2}}{Z_L} \\
&= \frac{Z_0^{\,2}}{Z_2} \\
&= \frac{1.5^2}{10 - j2} \\
&= 0.21 + j0.043
\end{aligned}
$$

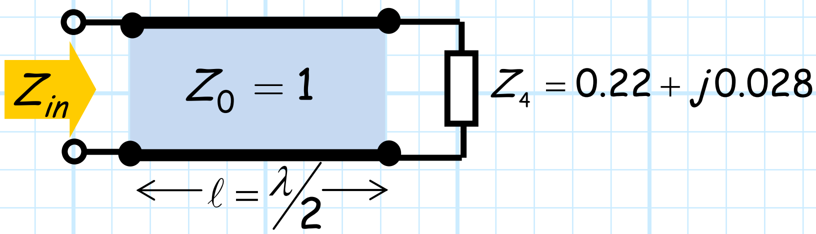
Thus, we can further **simplify** the original circuit as:



Now we find that the impedance Z_3 is **parallel** to the capacitor. We can **combine** the two impedances and define the result as impedance Z_4:

$$
\begin{aligned}
Z_4 &= -j3 \,\|\, (0.21 + j0.043) \\
&= \frac{-j3(0.21 + j0.043)}{-j3 + 0.21 + j0.043} \\
&= 0.22 + j0.028
\end{aligned}
$$

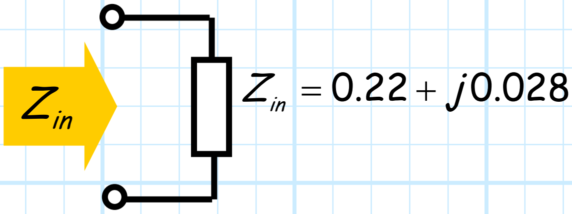
Now we are left with **this** equivalent circuit:



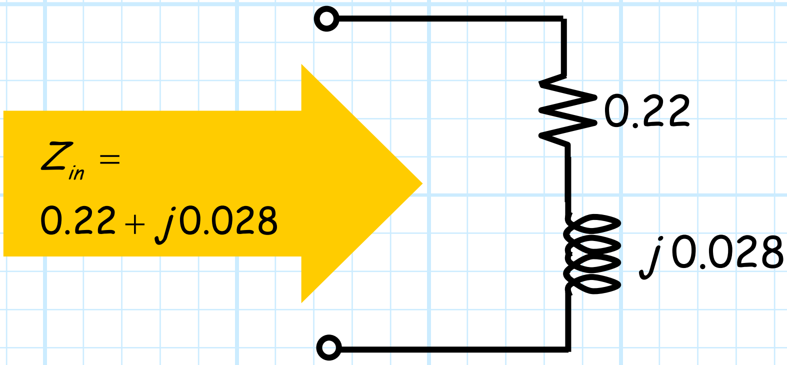
Note that the remaining transmission line section is a **half wavelength**! This is one of the **special** situations we discussed in a previous handout. Recall that the **input** impedance in this case is simply equal to the **load** impedance:

$$Z_{in} = Z_L = Z_4 = 0.22 + j0.028$$

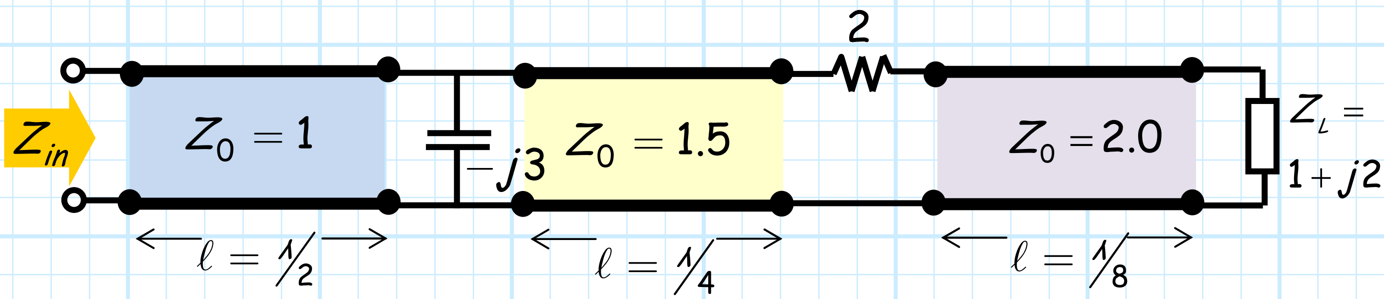
Whew! We are **finally** done. The **input impedance** of the original circuit is:



Note this means that **this** circuit:



and **this** circuit:



are precisely the **same**(at frequency ω_0)!

They have **exactly** the same impedance, and thus they "behave" precisely the **same** way in any circuit (but **only** at frequency ω_0!).